DESIGN, MATHEMATICAL MODELING AND CONTROL OF AN ASYMMETRICAL ELECTRORHEOLOGICAL DAMPER

Klaus Holzmann, Wolfgang Kemmetmüller, Andreas Kugi [*] **and Michael Stork** [**]

[*] *Chair of System Theory and Automatic Control, Saarland University, PO Box 15 11 50, 66041 Saarbrücken, Germany, E-Mail: andreas.kugi@lsr.uni-saarland.de*
[**] *Fludicon GmbH, Landwehrstrasse 50, 64293 Darmstadt, Germany*

Abstract: This paper is concerned with the design, the mathematical modeling and the control of an electrorheological (ER) damper for automotive applications. A continuous variation and the desired asymmetrical behavior of the damping characteristics is achieved by an intelligent combination of an ER valve with a laminar damping orifice and check valves. The resulting construction is distinguished by a very compact design and its fail-safe characteristics in the case of a fault of the high-voltage amplifier. Furthermore, some design considerations for the ER damper are given. Based on a detailed mathematical model a nonlinear control strategy is developed. Finally, the feasibility and the usefulness of the proposed construction and the nonlinear control strategy is demonstrated by means of simulation studies.

Keywords: electrorheological fluid, electrorheological damper, nonlinear control, semi-active suspension

1. INTRODUCTION

In the recent years increasing demands on driving stability and driving comfort necessitated the development of new active and semi-active suspension systems for automotive applications. This work deals with the design, the mathematical modeling and the control of an electrorheological (ER) damper which will be used in a semi-active suspension system for off-road vehicles. Passive dampers used in this field of application usually possess an asymmetrical damping characteristics, i.e. the damping is considerably higher for the extension than for the compression of the damper. This is mainly due to the fact that such a characteristics yields an optimum compromise between driving stability and driving comfort. An ER damper allows to actively influence the damping by means of an ER valve which is basically composed of two electrodes forming a flat gap with electrorheological fluid (ERF) flowing through. In general, an ERF is a suspension of polarizable particles in a fluid phase. The principle mode of operation is that the particles form chains or agglomerates in some manner upon application of a sufficiently large electric field strength. This in turn increases the apparent viscosity of the ERF and thus raises the pressure drop along the ER valve. Thereby, the formation of the agglomerates is very fast and thus makes the construction of ER dampers with high dynamics possible.

A number of works dealing with ER shock dampers (see, e.g. (Kugi *et al.*, 2005)) or active ER dampers (see, e.g. (Adams and Johnston, 2001), (Choi *et al.*, 2000), (Kim *et al.*, 2001)) have been published in the last years. Nonetheless, the ER dampers presented in these works exhibit a symmetrical damping characteristics and therefore are not perfectly suited to automotive suspension systems. The basic idea in the construction of the active ER damper presented in this work was to adopt the asymmetrical behavior of a passive automotive damper and to combine it with an active adjustment of the damping characteristics by means of an ER valve. Therein, the asymmetrical behavior of the ER damper is generated by combining the ER valve with a laminar damping orifice and check valves. The main advantages of this approach are the fail-safe function in case of a fault (e.g. of the high-voltage amplifier of the ER valve) and the fact that the ER damper can be designed in a very compact manner.

The paper is organized as follows: Section 2 presents the construction and the mathematical modeling of the ER damper. Furthermore, some design considerations for the ER damper are included in this section. Section 3 deals with the design of a nonlinear control strategy to enforce a desired damping characteristics. The usefulness and the feasibility of the proposed ER damper and the corresponding control strategy is shown by means of simulation results in Section 4. A conclusion and short outlook to further research activities closes this paper.

2. ELECTRORHEOLOGICAL DAMPER

2.1 Principal Concept

The ER damper depicted in Fig. 1 was designed for off-road vehicles. For this application the following two requirements are of essential importance: (i) the damping of the ER damper has to be continuously adjustable between a minimal and a maximal value at a high dynamics. Thereby, these values are different in the case of compression and extension of the ER damper due to the desired asymmetrical behavior. (ii) The ER damper should be designed in a compact form to allow for an easy assembly. The second requirement advises the usage of a differential cylinder with effective piston areas A_1 and A_2. To account for the difference $(A_1 - A_2)\, w$ of the displacement volume flows due to a velocity w of the piston a hydraulic accumulator is necessary. The accumulator is directly integrated in the damping cylinder in form of a piston-type accumulator, cf. Fig. 2.

The ER valve used for the active adjustment of the damping uses the wall of the cylinder as the inner

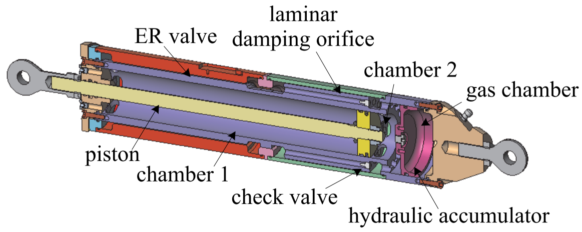

Fig. 1. Cross-section of the ER damper.

electrode which yields a very compact integration of the ER valve into the ER damper. To account for the asymmetrical damping characteristics, the ER valve is connected in series with a laminar orifice and a number of check valves, see the schematics in Fig. 2. With no voltage applied to the ER valve this yields a minimum damping of the ER damper in case of an extension of the damper. In case of a compression the damping orifice is bypassed by the check valves, whereby the pressure drop and with it the minimum damping is considerably lowered. In both cases the damping can be increased by applying a voltage to the ER valve. Both, the laminar damping orifice and the check valves are integrated in the walls of the damping cylinder, cf. Fig. 1. Of course, the asym-

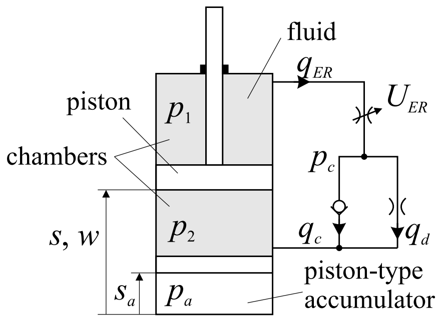

Fig. 2. Schematic diagram of the ER damper.

metrical behavior could also be generated by using only the ER valve. Nonetheless, the proposed construction has two major advantages: (i) using only the ER valve to cover the whole asymmetrical damping characteristics necessitates a much larger ER valve, which no longer could be easily integrated in the ER damper. Besides that, a larger ER valve also consumes more electrical power whereby larger high-voltage amplifiers might become necessary. (ii) In the case of a failure either of the controller or the high-voltage amplifiers the proposed concept still provides an asymmetrical damping characteristics and therefore, the vehicle can continue traveling, maybe at a reduced speed.

2.2 Mathematical Modeling

This subsection deals with the mathematical modeling of the ER damper. The mathematical model is based on the following observations: (i) in the ERF, the forces due to inertia are considerably lower than the forces due to viscosity and thus will be neglected in the sequel. (ii) The flows in the system are laminar. (iii) The ER effect only occurs inside the ER valve. In the rest of the ER damper the ERF can be treated as a viscous fluid. (iv) Due to the motion of the ER damper the ERF is heated up. Since the mathematical model is derived for the description of processes which are considerably faster than the dynamics of the temperature, we assume the temperature of the ERF to be constant.

The essential component of the ER damper is the ER valve. It is comprised of an inner electrode (cylinder of radius R_i), connected to earth, and an outer electrode (cylinder of radius R_o), connected to the voltage U, forming an annular gap (see Fig. 3). Since the height $H_{ER} = R_o - R_i$ of the gap is small compared to the mean radius $R_m = (R_o + R_i)/2$, the ER valve can be regarded as a flat channel with the length L_{ER} and the width $W_{ER} = 2R_m\pi$. In this work we will only give a short summary of the mathematical modeling of the ER valve. A detailed derivation of the results given here can be found in (Kemmetmüller and Kugi, 2004) and the references cited therein. The stationary flow in the ER valve is described by the following PDE

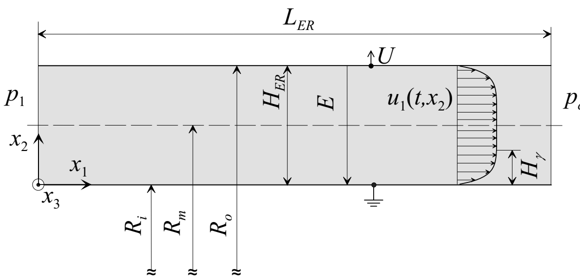

Fig. 3. Cross-section of the ER valve.

$$\frac{\partial \sigma_{12}}{\partial x_2} = P, \tag{1}$$

where σ_{12} denotes the shear stress, $P_{ER} = (p_1 - p_c)/L_{ER}$ is the pressure gradient, p_1 the pressure in the chamber 1 of the cylinder and p_c the pressure in the connection between the ER valve and the laminar damping orifice, cf. Fig. 2. The behavior of the ERF is described by a constitutive law of the form

$$\begin{aligned} \sigma_{12} &= \tau_0(E)\operatorname{sign}(\gamma) + \eta\gamma && \text{for } |\sigma_{12}| > \tau_0 \\ \gamma &= 0 && \text{else} \end{aligned} \tag{2}$$

with the electric field strength $E = U/H_{ER}$, the field dependent yield stress $\tau_0(E)$ and the dynamic viscosity of the ERF at zero electrical field strength η (see, e.g., (Rajagopal and Wineman, 1992), (Růžička, 2000) and (Eckart, 2000) for a detailed treatment of modeling of ERFs in the context of continuum mechanics). Furthermore, $\gamma = \partial u_1/\partial x_2$ denotes the shear rate and $u_1(t, x_2)$ the velocity, cf. Fig. 3. Microscopic calculations assuming an ideal dipole-dipole interaction between the particles of the ERF suggest that τ_0 has a quadratic growth with the electric field strength E (Parthasarathy and Klingenberg, 1996). Experiments have shown that for higher electrical fields a kind of saturation occurs, which is accounted for in the relationship $\tau_0(E) = \alpha_2 E^2 + \alpha_3 E^3$ with the parameters α_2 and α_3 which are determined experimentally (Kemmetmüller and Kugi, 2004).

The solution of (1) and (2) using the boundary conditions $u_1(0) = 0$ and $u_1(H_{ER}) = 0$ yields the stationary velocity profile in the ER valve. For vanishing electric field strength this profile is parabolic. With increasing electric field strength an increasing plug zone occurs in the middle of the gap, i.e. $\gamma = 0$ for $H_\gamma \le x_2 \le H_{ER} - H_\gamma$. If $P_{ER} < 2L_{ER}\tau_0(E)$, the plug zone covers the whole gap and the volume flow through the ER valve vanishes, i.e. $q_{ER} = 0$. In the other cases, the volume flow through the ER valve can be calculated by integration of the velocity profile over the gap height to

$$q_{ER} = \frac{W_{ER}(P_{ER}H_{ER} + \tau_0)(P_{ER}H_{ER} - 2\tau_0)^2}{12P_{ER}^2\eta} \tag{3}$$

The laminar damping orifice is also built up in form of an annular gap of height H_d, length L_d and diameter D_d. The same arguments used for the modeling of the ER valve justify the approximation of the annular gap in form of a rectangular gap of width $W_d = D_d\pi$. Thus, the laminar volume flow through the damping orifice yields (Blackburn *et al.*, 1960)

$$q_d = \frac{W_d H_d^3}{12\eta}\frac{\Delta p_d}{L_d} \tag{4}$$

with $\Delta p_d = p_c - p_2$ where p_2 denotes the pressure in the cylinder chamber 2.

An exact analytical model of the check valves is difficult to obtain since the geometry of the flow path is rather complicated. Therefore, we model the characteristics of the check valves in the form of a continuous function given by

$$q_c = \begin{cases} 0 & , \ \Delta p_d < p_{c,min} \\ b_0 + b_1\Delta p_d & , \ p_{c,min} \le \Delta p_d < p_{c,max} \\ c_0 + c_1\Delta p_d & , \ p_{c,max} \le \Delta p_d \end{cases} \tag{5}$$

where the coefficients b_i, c_i, $i = 0, 1$ and $p_{c,min}$, $p_{c,max}$ are determined by a least squares approximation of the measurement data provided by the manufacturer (Lee Company, 2005).

Neglecting the compressibility of the ERF in the connection of the ER valve with the damping orifice and the check valves, we can write the continuity equation by using the relations (3)–(5) in the form $q_{ER} = q_d + q_c$. With this, we can eliminate the pressure p_c and find the volume flow through the ER valve q_{ER} as a function of the difference of the chamber pressures $p_1 - p_2$ and the applied voltage U

$$q_{ER} = f_{ER}\left(p_1 - p_2, U\right). \tag{6}$$

Up to now, we have neglected the compressibility of the ERF. This assumption no longer holds for the derivation of the equations for the pressures p_1 and p_2 in the two chambers of the cylinder. Using the continuity equation for chamber 1, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}p_1 = \frac{\beta}{V_{01} - A_1 s}\left(A_1 w - \frac{q_{ER}\rho_0}{\rho_1}\right), \tag{7}$$

where $\beta = \rho_j \partial p_j / \partial \rho_j$, $j = 1, 2$, denotes the constant bulk modulus, V_{01} the effective volume for $s = 0$ and A_1 the effective piston area of chamber 1. Furthermore, the mass density of the ERF in the chamber 1 is given by $\rho_1 = \rho_0 \exp\left(p_1/\beta\right)$, with the mass density ρ_0 at ambient pressure.

The derivation of the equation for the pressure in chamber 2, which is connected to the hydraulic accumulator, is slightly more difficult. The thermodynamic process inside the accumulator can be regarded as isentropic since the motion of the piston is fast compared to the dynamics of the heat transfer. Thus, the gas in the accumulator can be described by the following equation

$$p_a V_a^{\kappa} = p_{0a} V_{0a}^{\kappa} = c_0. \tag{8}$$

Here, $\kappa = 1.6$ denotes the constant isentropic coefficient, p_a the pressure inside the accumulator and V_a the volume of the accumulator. Furthermore, the pre-charge condition of the accumulator is defined by the pressure p_{0a} and the volume V_{0a}, and is summarized by the constant c_0. The gas chamber of the accumulator is separated from the chamber 2 of the cylinder by a piston. Due to the facts that the mass of this piston is considerably small and that the friction occurring between this piston and the walls of the cylinder is very low we can neglect the influence of the piston. Thus, the accumulator pressure p_a is assumed to be equal to the chamber pressure p_2.

Now, the mass of ERF stored in the chamber 2 is given by

$$m_2 = \left(V_0 + A_2 s - V_a\right)\rho_2, \tag{9}$$

where ρ_2 is defined accordingly to ρ_1 by $\rho_2 = \rho_0 \exp\left(p_2/\beta\right)$ and $V_0 = V_{02} + V_{0a}$. The equation of continuity reads as

$$\frac{\mathrm{d}}{\mathrm{d}t}m_2 = \frac{\partial m_2}{\partial s}w + \frac{\partial m_2}{\partial p_2}\dot{p}_2 = \rho_0 q_{ER} \tag{10}$$

and results after some calculation in

$$\frac{\mathrm{d}}{\mathrm{d}t}p_2 = \frac{\kappa p_2 \beta\left(-A_2 w + \frac{q_{ER}\rho_0}{\rho_2}\right)}{\left(\frac{c_0}{p_2}\right)^{\frac{1}{\kappa}}\left(\beta - \kappa p_2\right) + \kappa p_2\left(V_0 + A_2 s\right)} \tag{11}$$

which completes the mathematical model of the ER damper.

For the subsequent controller design we simplify the model by neglecting the compressibility of the ERF, i.e. $\beta \to \infty$. With this we have $\rho_0 = \rho_1 = \rho_2$ and (7) reduced to $q_{ER} = A_1 w$ and (11) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}p_2 = \frac{\kappa p_2\left(-A_2 w + A_1 w\right)}{\left(\frac{c_0}{p_2}\right)^{\frac{1}{\kappa}}}. \tag{12}$$

To determine the pressure p_1 inside the chamber 1 we solve (6) for $q_{ER} = A_1 w$ and get

$$p_1 = p_2 + \Delta p\left(A_1 w, U\right). \tag{13}$$

For the investigation of the damping behavior we use the simple mechanical system depicted in Fig. 4. It is composed of a mass m which

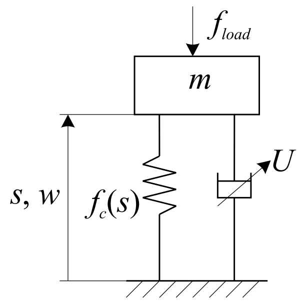

Fig. 4. System used in the simulation studies.

is supported by a spring (force $f_c\left(s\right)$) and the ER damper. An external load force f_{load} is the excitation of the system. The equation for this system read as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}s &= w \\ \frac{\mathrm{d}}{\mathrm{d}t}w &= \frac{1}{m}\left(-f_c\left(s\right) + \left(p_2 A_2 - p_1 A_1\right) - f_{load}\right). \end{aligned} \tag{14}$$

2.3 Design Considerations

In this subsection we will briefly give some insight into the choice of the parameters of the ER damper during the construction. The starting point in the design of the ER damper is the definition of the maximal value of the damping force $f_{d,max}$ which determines the diameter of the

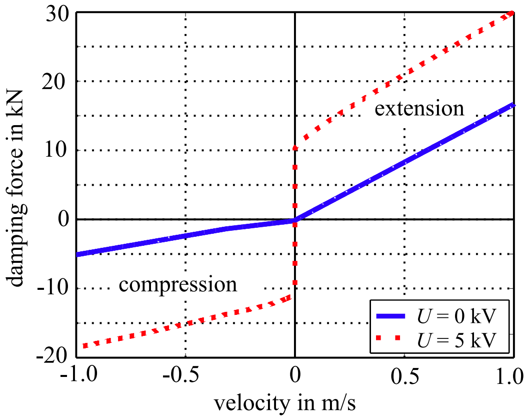

Fig. 5. Characteristic curve of the ER damper.

piston rod. Furthermore, the height H_{ER} of the ER valve is defined based on the maximum electric field strength. For the choice of the diameter of the cylinder the following considerations have to be taken into account: (i) the pressure drop which can be generated by an ER valve is basically proportional to the length L_{ER} of the ER valve. For a compact design the length should be kept as short as possible. Of course, less pressure drop along the ER valve results in a larger diameter of the cylinder to assure the necessary maximal damping forces. (ii) A large diameter of the cylinder results in large volume flows across the ER valve. The desired minimum damping of the ER damper necessitates a large width W_{ER} of the ER valve which certainly contradicts the request for a compact design of the system. Obviously, the two considerations are contradictory and an optimum between both has to be found. In our application it turned out that a length to width ratio of $L_{ER}/W_{ER} = 2/3$ is optimal.

Given the geometry of the ER valve, we can design the laminar damping orifice in such a way that in the case of extension of the damper the desired minimum damping characteristics is achieved. Moreover, the check valves are chosen in a way that in case of compression of the damper the pressure drop along the check valves is considerable lower than the pressure drop along the ER valve.

Finally, the volume of the accumulator V_{0a} and the pre-charge pressure p_{0a} are chosen such that the accumulator can hold the difference volume $(A_2 - A_1)\, s_{max}$ due to a maximal displacement s_{max} and that the pressure p_2 never falls below the ambient pressure p_0. Figure 5 depicts the resulting characteristics of the ER damper. Therein, the solid line marks the minimum damping force (i.e. for $U = 0$) whereas the dotted line displays the maximum damping force for $U = 5$kV. Furthermore, the asymmetrical behavior of the ER damper can be seen nicely in this figure.

3. CONTROLLER DESIGN

The ER damper designed and modeled in Section 2 is going to be used in automotive applications. In this task, the damping characteristics should be actively adjusted to the actual driving condition of the vehicle like e.g. the vehicle velocity, the displacement and velocity of the damper, the condition of the road, as well as to user-defined inputs. The specification of the damping behavior of the ER damper in view of driving comfort and stability is the task of an outer control structure which is not the topic of this paper. The task of the controller designed in this work is to track a desired damping characteristics as good as possible by means of the control input U. The controller design is complicated by the fact that only the position s and the velocity w of the damper but neither the chamber pressures nor the damping force can be measured.

For the controller design let us start by writing down the energy H of the system depicted in Fig. 4 according to the simplified system (12)–(14). It is given by the sum of the kinetic energy of the mass, the energy stored in the spring and the energy stored in the accumulator.

$$H = \frac{1}{2} m w^2 + \int_0^s f(\xi)\, \mathrm{d}\xi + \frac{p_2 \left(\frac{c_0}{p_2}\right)^{\frac{1}{\kappa}}}{\kappa - 1} \tag{15}$$

The time derivative of the energy H along the trajectories of the system (12)–(14) is given by

$$\dot{H} = \Delta p\,(A_1 w, U)\, A_1 w - f_{load} w. \tag{16}$$

If we now want to enforce a desired damping $\hat{f}_d\,(w, \chi)$, where χ summarizes the variables representing the driving situation and the user input, the following identity must hold

$$\Delta p\,(A_1 w, U)\, A_1 = \hat{f}_d\,(w, \chi)\,. \tag{17}$$

Thus, the nonlinear control law can be given in the form

$$U = \Delta p^{-1} \left(\frac{\hat{f}_d\,(w, \chi)}{A_1}, A_1 w \right). \tag{18}$$

It should be noted that this includes the case of a position-dependent damping force (e.g. a cushion) via χ. Obviously, this control law ensures that the damping characteristics of the ER damper is equal to the desired damping characteristics provided that the mathematical model of the ER damper is exact.

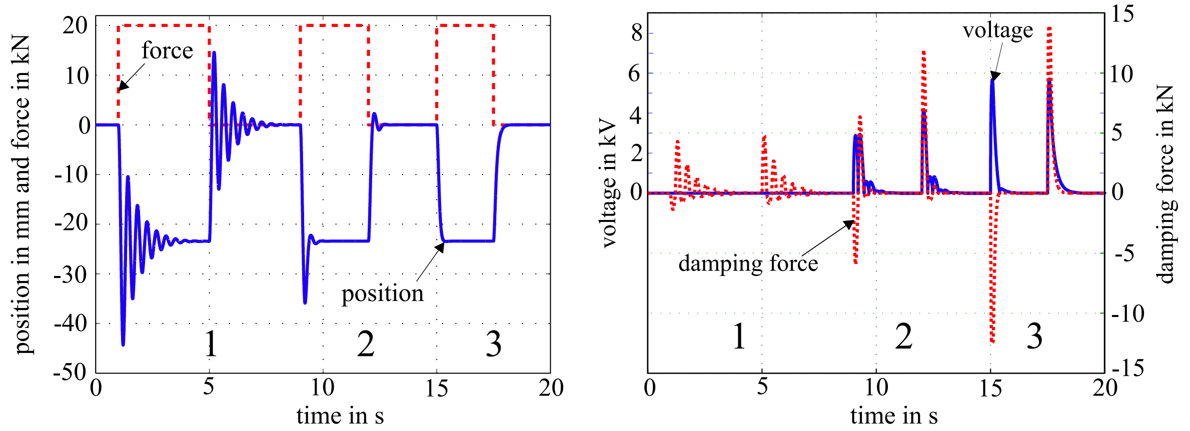

Fig. 6. Simulation results of the proposed control strategy.

4. SIMULATION STUDIES

To demonstrate the performance of the ER damper and the nonlinear control law (18) we show simulation results of the mechanical system depicted in Fig. 4. Therein, we choose a linear spring characteristics $f_c(s) = c_1 s$ with the constant stiffness of the spring c_1. The simulation is divided into three phases whereby the damping of the ER damper is controlled at its minimal value in phase 1 (according to $U = 0$). In the phases 2 and 3 the damping is controlled according to $\hat{f}_d = d_{comp} w$ for $w < 0$ and $\hat{f}_d = d_{ext} w$ for $w \geq 0$, where d_{comp} is the viscous damping coefficient for the compression and d_{ext} the viscous damping coefficient for the extension of the ER damper. In the second phase these coefficients are chosen to produce a medial damping behavior while in the third phase they are designed to produce a high damping characteristics. In both cases the asymmetrical behavior implicates $d_{comp} < d_{ext}$.

Figure 6 shows the simulation results of the step response of the system to a load force f_{load}. In the left part, the load force and the position s of the system is depicted. It can be easily seen that the damping of the system increases from phase 1 to 3. Furthermore, the asymmetrical behavior of the system can be identified in phase 2 where in the case of compression the position shows a much higher overshoot (due to less damping) than in the case of extension of the ER damper. This behavior is even more obvious if one has a look at the damping force $\Delta p A_1$, cf. (17), given in the right part of Fig. 6. Finally, the control input U is also depicted in the right part of Fig. 6.

5. CONCLUSION

In this contribution we presented the constructional design, the modeling and the controller design for an ER damper with asymmetrical behavior. Therein, the asymmetrical behavior was achieved by an intelligent combination of an ER valve with a laminar damping orifice and check valves. The advantages of this construction are the very compact design and the fail-safe behavior in case of a failure of the high-voltage amplifier of the ER valve. Based on a detailed mathematical model of the overall ER damper we gave some ideas of the choice of the parameters. Furthermore, we designed a nonlinear controller for tracking a desired, and within the limits of the damper arbitrary, damping characteristics of the ER damper based on an analysis of the dissipated energy. Finally, the feasibility and the usefulness of the construction and the control strategy was demonstrated by means of simulation studies.

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